

Probing CP Violation in $\tau^- \rightarrow \nu(K\pi/K2\pi/3K/K3\pi)^-$ Decays

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Abstract

It had been suggested to probe CP violation in $\tau \rightarrow \nu K\pi$ decays with $K^0 - \bar{K}^0$ oscillation to produce $A_{\text{CP}}(\tau^- \rightarrow \nu K_S \pi^-) = (0.36 \pm 0.01)\%$. BaBar has found $A_{\text{CP}}(\tau^- \rightarrow \nu K_S \pi^- [\geq \pi^0]) = (-0.36 \pm 0.23 \pm 0.11)\%$ – i.e., 2.8 sigma difference with SM prediction. It is discussed, why one needs to probe $A_{\text{CP}}(\tau \rightarrow \nu K\pi)$, $A_{\text{CP}}(\tau \rightarrow \nu K2\pi)$ and $A_{\text{CP}}(\tau \rightarrow \nu K3\pi)$ *separately* to establish the ‘existence’ of New Dynamics and its ‘features’. It should be possible at SuperB & Super-Belle experiments.

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1 CP Violation in Leptonic Dynamics

Large CP asymmetries have been established in B_d transitions by BaBar, Belle and CDF experiments, and the first evidence has appeared for $A_{\text{CP}}(D^0 \rightarrow K^+ K^-) - A_{\text{CP}}(D^0 \rightarrow \pi^+ \pi^-)$ [1, 2]. While we have found that CKM dynamics give at least the leading source of CP violation in B transitions, it cannot contribute significantly to the ”matter” vs. ”anti-matter” asymmetry in ‘our’ universe.

New Dynamics (ND) have been found by neutrino oscillations with $\theta_{12}, \theta_{23}, \theta_{13} > 0$ [3, 4]. A *necessary* condition for generating CP violation there has been satisfied. It gives

a good chance for leptonic dynamics producing ‘matter’ vs. ‘anti-matter’ asymmetry as a ‘shadow’ effect of the ‘lepton’ vs. ‘anti-lepton’ asymmetry. Furthermore probing CP symmetry at the level of $\mathcal{O}(0.1\%)$ in $\tau \rightarrow \nu[K\pi/K2\pi]$ has roughly the same sensitivity of ND in the amplitude as searching for $\text{BR}(\tau \rightarrow \mu\gamma)$ at the level of 10^{-8} [5, 6]. For CP odd observables in a SM allowed decay are *linear* in a ND amplitude, while in SM forbidden ones the rates are *quadratic* in ND amplitudes:

$$\text{CP odd} \propto T_{\text{SM}}^* T_{\text{ND}} \quad \text{vs.} \quad \text{LFV} \propto |T_{\text{ND}}|^2 \quad (1)$$

Leptonic EDMs and CP asymmetries in μ decays have been probed with high sensitivities [6] and should be continued.

Now BaBar Collaboration has found some evidence for CP violation in τ decays [8]:

$$A_{\text{CP}}(\tau^- \rightarrow \nu K_S \pi^- [\geq \pi^0]) = (-0.36 \pm 0.23 \pm 0.11)\% \quad (2)$$

CP violation established in $K^0 - \bar{K}^0$ oscillations gives as predicted [9, 10] (whether it is given by CKM dynamics or not):

$$A_{\text{CP}}(\tau^- \rightarrow \nu K_S \pi^-) = (0.36 \pm 0.01)\% ; \quad (3)$$

i.e., there is a difference of 2.8 sigma between these two values.

There is some experimental sign of *global* CP violation in τ decays. However, global asymmetries are often much reduced. One needs to probe different final states – include three- and four-body ones to established its (or their) *existence* of ND. Furthermore it is crucial to determine its (or their) *features*. We should focus on transitions that are CKM suppressed in SM – like $\tau \rightarrow \nu[K + \pi's]$ – where one has a good chance to identify both the impact and features of ND with less ‘background’ from SM amplitudes.

One needs conceptual lessons to understand the basis of the observed data on $\tau^- \rightarrow \nu K_S \pi^- [\geq \pi^0]$ vs. $\tau^- \rightarrow \nu[K\pi]^-$, $\tau^- \rightarrow \nu[K2\pi]^-$, $\tau^- \rightarrow \nu[3K]^-$ and $\tau^- \rightarrow \nu[K3\pi]^-$:

- CP asymmetries in $\tau \rightarrow \nu[K_S + \pi's]$ are generated by measured $K^0 - \bar{K}^0$ oscillations with great accuracy. One can measure rates and CP violations in $\tau^- \rightarrow \nu[K\pi's]^-$ vs. $\tau^+ \rightarrow \bar{\nu}[K + \pi's]^+$ and to calibrate ratios of $\tau^- \rightarrow \nu[\pi's]^-$ vs. $\tau^+ \rightarrow \bar{\nu}[\pi's]^+$, where one expects that ND can hardly produce measurable asymmetries.
- For $\tau^- \rightarrow \nu[K\pi]^-$ one gets contributions mostly from $\tau^- \rightarrow \nu K^*(892)$ with some from $\tau^- \rightarrow \nu K_0^*(1430)$ due to vector and scalar exchanges.
- For $\tau \rightarrow \nu[K2\pi]/\nu[3K]/\nu[K3\pi]$ one has more CP odd observables through *moments* and their *distributions* to check the impact of ND. Those are described by total four- & five-body final states – and therefore *hadronic* three- & four-body final states with *distributions* of hadronic masses [5, 11, 12, 14]. In particular one should probe $K^*(892)\pi$, $K_1(1270)$, $K_1(1400)$ & $K^*(1410)$ hadronic final states and in particular their *interferences*.

- Beyond $K^0 - \bar{K}^0$ oscillations one probes *direct* CP violation in τ decays. Unless one has *longitudinally polarized* τ , one needs differences in both the weak and strong phases to generate CP asymmetries in $\tau \rightarrow \nu[K\pi]$. Non-zero T odd observables can be produced by FSI without CP violation. On the other hand true CP asymmetries can be probed for τ^- vs. τ^+ decays.
- CPT symmetry predicts

$$\Gamma(\tau^- \rightarrow \nu + [S = -1]) = \Gamma(\tau^+ \rightarrow \bar{\nu} + [S = 1]) \quad (4)$$

with

$$[S = -1] = \bar{K}^0\pi^-/K^-\pi^0/\bar{K}^0\pi^-\pi^0/K^-\pi^+\pi^-/K^-\pi^0\pi^0/ \quad (5)$$

$$K^-K^+K^-/K^-\bar{K}^0K^0/\bar{K}^0(3\pi)^-/K^-(3\pi)^0 \text{ etc.} \quad (6)$$

Two items have to be dealt with:

- One measures final states with K_S , K_L and the interferences between them. $K^0 - \bar{K}^0$ oscillation impacts CP asymmetries as expressed by $2\text{Re } \epsilon_K$ in a *global* way for channels.
- Mixing between $\bar{K}^0\pi^- \rightarrow K^-\pi^0$, $\bar{K}^0\pi^0 \rightarrow K^-\pi^+$ and $K^-K^+ \rightarrow K^0\bar{K}^0$ happen by FSI. Diagrams show it, but we cannot control it quantitatively.

Therefore one can learn crucial lessons about the underlying dynamics by identifying those final states *separately*. The branching ratios of these transitions are not small [15].

2 CP Asymmetry in $\tau^- \rightarrow \nu[K\pi]^-$

One has three-body final states with two hadrons h_1 & h_2 with variation in $M^2(h_1h_2)$ with vector and scalar resonances. CP asymmetries depend on different weak and strong phases.

Final states $K\pi$ are produced from the QCD vacuum with vector and scalar configurations with form factors F_V and F_S [16]; the vector component is dominated mainly in the form of K^* . In principle the latter produces no problem, since several resonances contribute at different mass values. In the SM one gets no different weak phase from quark and lepton dynamics – however ND can generate different weak phases due exchanges of charged Higgs or the ‘old standby’ for enhanced ND effects, namely SUSY with broken R parity. Amplitudes with scalar resonances are suppressed. Therefore their contributions are hardly to be found for total widths; however they can generate *interference* with vector resonances with ‘local’ CP asymmetries up to of $\mathcal{O}(\%)$ [17]. Therefore one has to probe the ‘topology’ of the three-body final states for $\tau^- \rightarrow \nu K_S\pi^-$ vs. $\tau^+ \rightarrow \bar{\nu} K_S\pi^+$ [18] and $\tau^- \rightarrow \nu K^-\pi^0$ vs. $\tau^+ \rightarrow \bar{\nu} K^+\pi^0$ by $d\Gamma/dE_K$ or $d\Gamma/dM_{K\pi}$ etc.

The ‘Miranda procedure’ has been suggested for three-body final states for B and D decays for *localizing* CP asymmetries in Dalitz plots [11, 12]. It can be applied here

independent of τ production asymmetry with plots of E_K vs. $M_{K\pi}$. In particular one can compare regions with positive and negative interference between vector and scalar states for τ^- vs. τ^+ , which gives significant lessons on the underlying ND. ‘Miranda procedure’ is based on analyzing the *significance*

$$\Sigma(i) \equiv \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}} \quad (7)$$

in the final state plot rather the customary *fractional* asymmetry

$$\Delta(i) \equiv \frac{N(i) - \bar{N}(i)}{N(i) + \bar{N}(i)} . \quad (8)$$

At a SuperB experiment proposed and approved near Rome in Italy one could produce a pair of longitudinally polarized τ and therefore probe T *odd* moments and their distributions in $\tau \rightarrow \nu h_1 h_2$ decays.

3 CP Violation in $\tau \rightarrow \nu h_1 h_2 h_3 / \nu h_1 h_2 h_3 h_4$

Final states with three or four hadrons in the final state produce many more CP sensitive observables. Therefore we have more information about the existence and the features of the ND and check also experimental uncertainties [5, 6]. In the SM one gets zero CP asymmetries in $\tau^- \rightarrow \nu K^- [S=0]^0$ and only a global one in $\tau^- \rightarrow \nu K_S [S=0]^-$ due to $2\text{Re}(\epsilon_K)$. ND in those decays has to compete only with SM Cabibbo suppressed ones.

3.1 CP Asymmetry in $\tau^- \rightarrow \nu [K2\pi]^- / \nu [3K]^-$

For $A_{\text{CP}}(\tau^- \rightarrow \nu K^- \pi^+ \pi^-)$ one predicts global zero CP asymmetry in SM and $(0.36 \pm 0.01)\%$ as before due to $K^0 - \bar{K}^0$ oscillation for $A_{\text{CP}}(\tau^- \rightarrow \nu K_S \pi^- \pi^0)$. A richer landscape for ND can surface in $\tau \rightarrow \nu K 2\pi$ due to contributions from $K^* \pi$, $K \sigma$, $\kappa \pi$ etc., where one sees triple-product asymmetries [7]. To be more practical: One can measure *T odd moments* $\langle \vec{p}_K \cdot (\vec{p}_{\pi_1} \times \vec{p}_{\pi_2}) \rangle$ for τ^- vs. τ^+ decays.

One can also probe their Dalitz plots with one refinement: the total mass of the hadronic final state is not fixed – it depends on the energy of the neutrino. One can follow the qualitative example given in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ transitions [13] (and suggested for $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ [5]). Final states with three hadrons produce huge fields for CP observables even for unpolarized τ leptons: their Dalitz plots can be probed depending on the energy of the neutrino. This can be seen as an ‘excess of riches’. However one has to think which observables give us the ‘best’ lessons about the underlying dynamics in the ‘real’ world. For example, one can focus on measuring the angle between the plane of the two hadrons and the plane of the neutrino and the third hadron in the τ rest frame – like $\pi^+ - \pi^-$ and $\nu - K^-$:

$$\frac{d}{d\Phi_{+-}} \Gamma(\tau^- \rightarrow \nu K^- \pi^+ \pi^-) = \Gamma_1^{K^-} \cos^2 \Phi_{+-} + \Gamma_2^{K^-} \sin^2 \Phi_{+-} + \Gamma_3^{K^-} \cos \Phi_{+-} \sin \Phi_{+-} \quad (9)$$

$$\frac{d}{d\Phi_{-+}}\Gamma(\tau^+ \rightarrow \bar{\nu}K^+\pi^-\pi^+) = \bar{\Gamma}_1^{K^+}\cos^2\Phi_{-+} + \bar{\Gamma}_2^{K^+}\sin^2\Phi_{-+} + \bar{\Gamma}_3^{K^+}\cos\Phi_{-+}\sin\Phi_{-+} \quad (10)$$

Using $\Phi_{+-} = -\Phi_{-+}$, $\cos^2\Phi_{+-} = \cos^2\Phi_{-+}$, $\sin^2\Phi_{+-} = \sin^2\Phi_{-+}$, $\cos\Phi_{+-}\sin\Phi_{+-} = -\cos\Phi_{-+}\sin\Phi_{-+}$ one gets:

$$\Gamma(\tau^- \rightarrow \nu K^-\pi^+\pi^-) - \Gamma(\tau^+ \rightarrow \bar{\nu}K^+\pi^-\pi^+) = \frac{\pi}{2} \left([\Gamma_1 - \bar{\Gamma}_1] + [\Gamma_2 - \bar{\Gamma}_2] \right) \quad (11)$$

$$\int_0^{\pi/2} d\Phi_{+-} (\Gamma_{\tau^-} + \Gamma_{\tau^+}) - \int_{\pi/2}^{\pi} d\Phi_{+-} (\Gamma_{\tau^-} + \Gamma_{\tau^+}) = \Gamma_3 + \bar{\Gamma}_3; \quad (12)$$

i.e., ‘global’ CP asymmetry $\Gamma(\tau^- \rightarrow \nu K^-\pi^+\pi^-) \neq \Gamma(\tau^+ \rightarrow \bar{\nu}K^+\pi^-\pi^+)$ and the first step towards ‘local’ CP violation.

The next step for localizing CP asymmetry is to measure

$$\Gamma_1 \neq \bar{\Gamma}_1, \Gamma_2 \neq \bar{\Gamma}_2, \Gamma_3 \neq \bar{\Gamma}_3. \quad (13)$$

Strong QCD forces can generate $0 \neq \Gamma_3 = -\bar{\Gamma}_3$; however CP violation shows $\Gamma_3 + \bar{\Gamma}_3 \neq 0$. Measuring $\Gamma_{1,2,3}$ & $\bar{\Gamma}_{1,2,3}$ *separately* with $\cos^2\Phi_{+-}$, $\sin^2\Phi_{+-}$ and $\cos\Phi_{+-}\sin\Phi_{+-}$ also help experimental uncertainties.

Furthermore one can measure the angles between the planes of $K^- - \pi^+$ and $\nu - \pi^-$ vs. $K^+ - \pi^-$ and $\bar{\nu} - \pi^+$ or $K^- - \pi^-$ and $\nu - \pi^+$ vs. $K^+ - \pi^+$ and $\bar{\nu} - \pi^-$ in τ^- and τ^+ decays. Those angles $\Phi_{K^-\pi^+}$ vs. $\Phi_{K^+\pi^-}$ or $\Phi_{K^-\pi^-}$ vs. $\Phi_{K^+\pi^+}$ tell us more of the underlying dynamics in $\tau \rightarrow \nu K \pi \pi$ transitions.

Likewise one can probe $\tau^- \rightarrow \nu K^- K^+ K^-$ vs. $\tau^+ \rightarrow \bar{\nu} K^+ K^- K^+$. It is more challenges for $\tau^- \rightarrow \nu K^- \pi^0 \pi^0$ vs. $\tau^+ \rightarrow \bar{\nu} K^+ \pi^0 \pi^0$ and $\tau^- \rightarrow \nu K_S \pi^- \pi^0$ vs. $\tau^+ \rightarrow \bar{\nu} K_S \pi^+ \pi^0$.

As the final step of probe CP asymmetries one can measure the *distributions* of $\vec{p}_K \cdot (\vec{p}_{\pi_1} \times \vec{p}_{\pi_2})$ with $d\Gamma/dM_{K\pi}$ and $d\Gamma/dM_{K2\pi}$ or with $d\Gamma/dM_{2K}$ and/or $d\Gamma/dM_{3K}$.

As mentioned before the sizable number of kinematical variables and of specific channels allow more internal crosschecks of systematic uncertainties like detection efficiencies for positive vs. negative particles. In addition:

- ND can interfere *vector* with *axial vector* configurations. For example, ND could be based on W_R exchanges coupling to right-handed leptons and quarks.
- Quantitative correlations of $\Gamma_3^K - \bar{\Gamma}_3^K$ with $\Gamma_3^\pi - \bar{\Gamma}_3^\pi$ are important. These should be possible at SuperB/Super-Belle.

‘Miranda Procedure’ can be applied as mentioned above; it is driven by data for ‘partitioning’ the Dalitz plots. However one needs some refinement: The $K2\pi$ and $3K$ masses are not fixed – they depend on the neutrino kinematics impact. Dividing the hadronic mass spectrum into two or three parts could help significantly depending on future data.

3.2 CP Asymmetry in $\tau^- \rightarrow \nu[K3\pi]^-$

The landscape is even richer for impact of ND for these final states from $K^*\rho$, $K^*\sigma$, $K\omega$, $\kappa\rho$ etc. There are one several different T odd moments for $\tau^- \rightarrow \nu K^- \pi^+ \pi^- \pi^0$

$$\langle \vec{p}_{K^-} \cdot (\vec{p}_+ \times \vec{p}_-) \rangle, \quad \langle \vec{p}_{K^-} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^0}) \rangle \quad \text{etc.} \quad (14)$$

and for $\tau^- \rightarrow \nu K_S \pi^- \pi^+ \pi^-$

$$\langle \vec{p}_{K_S} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \rangle, \quad \langle \vec{p}_{K_S} \cdot (\vec{p}_{\pi^-} \times \vec{p}_{\pi^-}) \rangle \quad \text{etc.} \quad (15)$$

and for distributions in $M_{h_1 h_2}$, $M_{h_1 h_2 h_3}$ and $M_{h_1 h_2 h_3 h_4}$. Of course it will need more experimental work – but it would tell us more of the features of ND.

4 Summary

SM cannot generate measurable CP asymmetries in $\tau^- \rightarrow \nu[K^- + \pi's]$ and a value of $(0.36 \pm 0.01)\%$ in total widths for $\tau^- \rightarrow \nu[K_S + \pi's]$. ND – like with charged Higgs or W_R exchanges – can affect these decays with hadronic two-, three- and four-body final states significantly with probing regions of interference between different resonances. To be more precise:

- One has to compare $\Gamma(\tau^- \rightarrow \nu[K\pi]^-)$ vs. $\Gamma(\tau^+ \rightarrow \bar{\nu}[K\pi]^+)$, $\Gamma(\tau^- \rightarrow \nu[K2\pi]^-)$ vs. $\Gamma(\tau^+ \rightarrow \bar{\nu}[K2\pi]^+)$, $\Gamma(\tau^- \rightarrow \nu[3K]^-)$ vs. $\Gamma(\tau^+ \rightarrow \bar{\nu}[3K]^+)$ and $\Gamma(\tau^- \rightarrow \nu[K3\pi]^-)$ vs. $\Gamma(\tau^+ \rightarrow \bar{\nu}[K3\pi]^+)$.
- As emphasized before about B and D decays with three- and four-body final states, one gets contributions from resonances and their interferences for CP asymmetries. However ‘global’ asymmetries averaged over the total widths are significantly smaller than individual contributions.
- Therefore it is very important to probe the ‘topology’ in the Dalitz plots.
- For $\tau^- \rightarrow \nu[K\pi]^-$ one can probe interference between vector and scalar states, which are somewhat suppressed. For $\tau^- \rightarrow \nu[K2\pi]^-/[3K]^-$ one can probe T odd moments due to vector and axial vectors exchanges and even more for $\tau^- \rightarrow \nu[K3\pi]^-$, which should be not suppressed in general.
- On the step to probe the final states as discussed above one can look for *local* asymmetries in $\tau^- \rightarrow \nu[3K + K2\pi]^-$ vs. $\tau^+ \rightarrow \bar{\nu}[3K + K2\pi]^+$.

SuperB and Super-Belle experiments should be able to probe the *whole* area of $\tau \rightarrow \nu[K\pi/K2\pi/3K/K3\pi]$ transitions with *neutral* pions in the final states.

One more comment about CP asymmetries in τ decays: These comments about the impact of ND is focused on semi-hadronic τ transitions. It is most likely to affect also B

and D decays – but it could ‘hide’ more easily there due to larger effects – in particular for B transitions – and less control over non-perturbative QCD effects.

A last comment: I have emphasized to probe the distributions of final states in τ (and B/D) decays to find the impacts and the features of ND(s) based on ‘binned’ [11, 12] and ‘unbinned multivariate’ [14] results.

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